

Mathematical Modeling of Insurance Mechanisms for E-commerce Systems

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Abstract

Electronic commerce (a.k.a. E-commerce) systems such as eBay and Taobao of Alibaba are becoming increasingly popular. Having an effective reputation system is critical to this type of internet service because it can assist buyers to evaluate the trustworthiness of sellers, and it can also improve the revenue for reputable sellers and E-commerce operators. We formulate a stochastic model to analyze an eBay-like reputation system and propose four measures to quantify its effectiveness: (1) new seller ramp up time, (2) new seller drop out probability, (3) long term profit gains for sellers, and (4) average per seller transaction gains for the E-commerce operator. Through our analysis, we identify key factors which influence these four measures. We propose a new insurance mechanism which consists of an insurance protocol and a transaction mechanism to improve the above four measures. We show that our insurance mechanism can reduce the ramp up time by around 87.2%, and guarantee new sellers ramp up before the deadline T_w with a high probability (close to 1.0). It also increases the long term profit gains and average per seller transaction gains by at least 95.3%.

1 Introduction

E-commerce systems are becoming increasingly popular and typical examples include eBay[9], Amazon[1], and Taobao [22] of Alibaba (the largest E-commerce system in China), etc. Through an E-commerce system, geographically distributed sellers and buyers can transact online. Sellers advertise products in their online stores (which reside in the E-commerce's website), while buyers can purchase products from any online stores. The E-commerce system charges a transaction fee from sellers for each completed transaction. In an E-commerce system, it is possible to purchase products from a seller whom the buyer has never transacted with, and this seller may not even be trustworthy[19]. This situation results in a high risk of buying low quality products. To overcome such problems, E-commerce systems deploy reputation systems[19].

Usually, E-commerce operators maintain and operate a reputation mechanism to reflect the *trustworthiness* of sellers[9, 22]. A high reputation seller can attract more transactions leading to higher revenue[19]. The eBay-like reputation system is the most widely deployed reputation policy, which is used in eBay and Tabao, etc. This type of reputation system is a credit based system. More precisely, a seller needs to collect enough credits from buyers in order to improve his reputation. These credits are obtained in form of feedback ratings, which are expressed by buyers after each transaction. Feedback ratings in eBay and Taobao are of three levels: positive (+1), neutral (0), and negative (-1). The cumulative sum of all the past feedback ratings (i.e., reputation score) reflects the *trustworthiness* of a seller. The reputation score and feedback ratings are public information and accessible by all buyers and sellers in such an E-commerce system.

Consider this eBay-like reputation system, a new seller may spend a long time to collect enough credits (i.e., ramp up). This is because new sellers are initialized with a reputation score of zero, and buyers are less willing to buy products from a seller with low reputation scores. The ramp up time is critical to the effectiveness of a reputation system. A long ramp up time discourages new sellers to join an E-commerce system. Furthermore, a new user starts an online store with certain budgets, and maintaining such online stores involves cost. If a new seller uses up his entire budget and has not yet ramped up his reputation, he may discontinue his online business (i.e, or drops out) due to low revenue. Therefore, a long ramp up time increases the risk that a new seller drops out and discourages potential new sellers to join. Finally, a long ramp up time also results in a low profit gain for a seller. Because before ramping up, a seller can only attract few transactions due to his low reputation score. To an E-commerce operator, this also results in an indirect loss on transaction gains. This paper aims to identify key factors that influence the ramp up time and to design a mechanism to improve this measure.

Reducing ramp up time is challenging and to the best of our knowledge, this is the first work which explores how to reduce the ramp up time for an eBay-like reputation system. This paper aims to explore the following fundamental questions: (1) *How to identify key factors which influence the ramp up time?* (2) *How to take advantage of these factors to reduce the ramp up time?* Our contributions are

- We propose four performance measures to quantify the effectiveness of eBay reputation systems: (1) new seller ramp up time, (2) new seller drop out probability, (3) long term profit gains for sellers, and (4) average per seller transaction gains for an E-commerce operator.
- We develop a stochastic model to identify key factors which many influence these four measures. Through we gain important insights on how to design a new mechanism these performance measures.
- We propose and design an insurance mechanism which can reduce the ramp up time and the new seller drop out probability. We show that our insurance mechanism can reduce the ramp up time by around 87.2%, and guarantee new sellers ramp up before the deadline T_w with a high probability (close to 1.0). It also increases the long term profit gains and average per seller transaction gains by at least 95.3%.

This paper organizes as follows. In §2, we present the system model for E-commerce systems. In §3 we formulate four measures to explore the ramp up time problem, i.e., ramp up time, new seller drop out probability, long term profit gains and average per seller transaction gains. In §4, we derive analytical expressions for these four measures. In §5 present the design of our insurance mechanism. Related work is given in §6 and we conclude in §7.

2 E-commerce System Model

An E-commerce system consists of *users*, *products* and a *reputation system*. A user can be a *seller* or a *buyer* or both. Sellers advertise products in their online stores and set a price for each product. Buyers, on the other hand, purchase products through online stores and provide feedbacks to indicate whether a buyer advertises products honestly or not. A reputation system is maintained by E-commerce operators to reflect the *trustworthiness* of sellers. A high reputation seller can attract more transactions leading to a high revenue. The reputation system aggregates all the feedbacks, and computes a reputation score for each seller. The reputation score is public information which is accessible by all buyers and sellers.

Products are categorized into different types. For example, eBay categorizes products into “Fashion”, “Electronics”, “Collectibles & Art”, etc [9]. We consider $L \geq 1$ types of product. Consider a type $\ell \in \{1, \dots, L\}$ product. A seller sets a price $p_\ell \in [0, 1]$ and the E-commerce operator charges a transaction fee of $T \triangleq \alpha p_\ell$, where $\alpha \in (0, 1)$, after the product is sold¹. It has a manufacturing cost of $c_\ell \in [0, 1]$. A seller earns a profit of u_ℓ by selling one product, we have

$$u_\ell = (1 - \alpha)p_\ell - c_\ell. \quad (1)$$

For the ease of presentation, our analysis focuses on one product type. It can be easily generalized to multiple product types, and we omit the subscript unless we state otherwise.

2.1 Transaction Model

Sellers advertise the product quality in their online stores. Let $Q_a \in [0, 1]$ be the *advertised quality*. The larger the value of Q_a implies the higher the advertised quality. Buyers refer to the advertised quality Q_a in their product adoption. Each online store also has an intrinsic quality. Let $Q_i \in [0, 1]$ be the *intrinsic quality* (i.e., the ground truth of the product’s quality). The larger the value of Q_i implies the higher the intrinsic quality. Since sellers aim to promote their products, so we have $Q_a \geq Q_i$. We emphasize that the intrinsic quality Q_i is private information, e.g., it is only known to the seller. On the other hand, the advertised quality Q_a is public information which is accessible by all buyers and sellers.

Buyers estimate the product quality by referring to the advertised quality Q_a (we will present the estimating model later). Let $Q_e \in [0, 1]$ be the *estimated quality*. The larger the value of Q_e implies the higher the estimated quality. To purchase a product, a buyer must submit a payment p to the E-commerce system, which will be given to the corresponding seller when he receives the product. There is usually a shipment delay in any E-commerce systems. We denote the delay as d . Upon receiving a product, a buyer can evaluate its quality and at that moment, he has the *perceived quality*, which we denote as $Q_p \in [0, 1]$. The larger the value of Q_p implies the higher the perceived quality. We assume that buyers can perceive the *intrinsic*

¹We can also consider a fixed transaction fee model and our analysis is still applicable. But for brevity, let us consider a transaction fee which is proportional to the selling price.

quality, i.e., $Q_p = Q_i$. Buyers are satisfied (disappointed) if they find out that the product is at least as good as (less than) it is advertised, or $Q_p \geq Q_a$ ($Q_p < Q_a$).

To attract buyers, an E-commerce system needs to incentivize sellers to advertise honestly, i.e., $Q_a = Q_i$. Many E-commerce systems achieve this by deploying a reputation system. We next introduce a popular reputation system used by many E-commerce systems such as eBay [9] or Taobao [22]. Table 1 summarizes key notations in this paper.

p, c	price and manufacturing cost of a product
T, u	transaction fee, unit profit of selling a product
Q_a, Q_i, Q_e, Q_p	advertised, intrinsic, estimated, perceived product quality
d, C_S	shipment delay, shipment cost
γ	critical factor in expressing feedback ratings
\mathcal{F}	reputation profile for a seller
r	reputation score for a seller
r_h, θ	reputation threshold, consistency threshold
β	discounting factor in estimating product quality
$\mathcal{P}(Q_e, p)$	probability that a buyer buys a product with an estimated quality Q_e and a price p
P_{ba}, P_{br}	probability that a buyer buys a product from a seller labelled as average (reputable)
$\lambda_1(\lambda_2)$	buyer's arrival rate before (after) a seller ramps up
T_w	the maximum time that a seller is willing to wait to get ramped up
T_r, P_d	ramp up time, new seller drop out probability
G_s, G_e	long term expected profit gains for a seller, average per seller transaction gains for the E-commerce operator
δ	discount factor in the long term expected profit gains G_s
$\lambda_T(\tau)$	transaction's arrival rate at time slot τ
C_I, D_I, T_d, T_c	insurance price, deposit, duration time and clearing time
\widehat{D}_I	insurance deposit threshold to revoke insurance certificate
λ_I	transaction's arrival rate to an insured seller
T_r^I, P_r^I	ramp up time with insurance, new seller drop out probability with insurance
G_s^I, G_e^I	long term expected profit gains with insurance, average per seller transaction gains with insurance

Table 1: Notation list

2.2 Baseline Reputation System

The eBay-like system maintains a reputation system to reflect the trustworthiness of sellers. It consists of a *feedback rating system* and a *rating aggregation policy*.

Buyers express feedback ratings to indicate whether a seller advertises honestly or not. The eBay-like system adopts a feedback rating system consisting of three rating points², i.e., $\{-1, 0, 1\}$. A positive rating (rating 1) indicates that a product is at least as good as it is advertised, i.e., $Q_p \geq Q_a$. A neutral rating (rating 0) indicates that a buyer is indifferent about the product that he purchased. This happens when the perceived quality is slightly lower than it is advertised, i.e., $Q_p \in [Q_a - \gamma, Q_a]$, where $\gamma \in [0, 1]$ denotes the critical factor. The smaller the value of γ implies that buyers are more critical in expressing ratings, e.g., $\gamma = 0$ means that buyers have zero tolerance on seller overstating the product quality. A negative rating (rating -1) represents that the perceived quality is far smaller than the advertised quality, i.e., $Q_p < Q_a - \gamma$. We have

$$\text{feedback rating} = \begin{cases} 1, & \text{if } Q_p \geq Q_a, \\ 0, & \text{if } Q_a - \gamma \leq Q_p < Q_a, \\ -1, & \text{if } Q_p < Q_a - \gamma. \end{cases}$$

All the historical ratings are known to all buyers and sellers.

²We can easily generalize the model to consider more rating points.

For the rating aggregation policy, each seller is associated with a reputation score, which is the summation of all his feedback ratings. We denote it by $r \in \mathbb{Z}$. A new seller who enters the E-commerce system is initialized with zero reputation score, or $r = 0$. A positive feedback rating increases r by one, a negative feedback rating decreases r by one, and a neutral feedback rating 0 does not change r . Figure 1 depicts the transition diagram of a seller's reputation score. Note that r is a public information accessible by all buyers and sellers.

To assist buyers to evaluate the trustworthiness of a seller, E-commerce systems not only announce the seller's reputation score r , but also his reputation profile. Let $\mathcal{F} \triangleq (r, n^+, n^0, n^-)$ be the reputation profile, where n^+, n^0, n^- represent the *cumulative number of feedback ratings* equal to 1, 0, -1 respectively. Note that this form of reputation is commonly deployed, say in eBay [9] and Taobao [22].

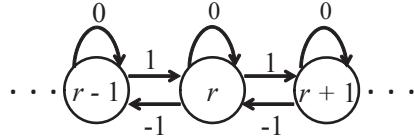


Figure 1: Transition diagram of a seller's reputation score r

Shipment delay in real-world E-commerce systems usually results in certain delay in the reputation update. To characterize the dynamics of a reputation updating process, we consider a discrete time system and divide the time into slots, i.e., $[0, d], [d, 2d], \dots$, where d is the shipment delay. We refer to a time slot $\tau \in \mathbb{N}$ as $[\tau d, (\tau + 1)d]$. Let $N(\tau)$ be the number of products sold in the time slot τ . Suppose $N^+(\tau), N^0(\tau), N^-(\tau)$ of these transactions result in positive, neutral and negative feedbacks respectively. Let $\mathcal{F}(\tau) \triangleq (r(\tau), n^+(\tau), n^0(\tau), n^-(\tau))$ be the reputation profile at time slot τ . Then we have $\mathcal{F}(0) = (0, 0, 0, 0)$. We update the reputation profile $\mathcal{F}(\tau)$ as

$$\begin{cases} n^+(\tau + 1) = n^+(\tau) + N^+(\tau), \\ n^0(\tau + 1) = n^0(\tau) + N^0(\tau), \\ n^-(\tau + 1) = n^-(\tau) + N^-(\tau), \\ r(\tau + 1) = r(\tau) + N^+(\tau) - N^-(\tau). \end{cases} \quad (2)$$

For simplicity, we drop the time stamp τ in the reputation profile, when no confusion involved.

We next present a model to characterize the impact of sellers' reputation profiles on buyers' product adoption behavior. This model serves as an important building block to explore the effectiveness of this baseline reputation system.

2.3 Model for Product Adoption Behavior

The reputation system forges trust among sellers and buyers. This trust plays a critical role in product adoption. More precisely, buyers evaluate the trustworthiness of sellers from sellers' reputation profiles. Buyers seek to minimize the risk in product purchase and they prefer to buy from reputable sellers.

Based on the reputation profile \mathcal{F} , our model classifies sellers into two types: “*reputable*” and “*average*”. To be labeled as reputable, a seller's reputation profile must satisfy two conditions. The first one is that a seller needs to collect enough credits, i.e., positive feedbacks from buyers. More precisely, his reputation score must be at least greater than or equal to some positive reputation threshold r_h , i.e., $r \geq r_h$. A new seller is initialized with zero reputation score, i.e., $r = 0$. To accumulate a reputation score of at least r_h , he needs to accomplish enough number of honest transactions. The second condition is that a seller should be *consistently honest*. More concretely, the fraction of positive feedbacks should be larger than or equal to a consistency threshold $\theta \in (0, 1]$, i.e., $n^+/(n^+ + n^- + n^0) \geq \theta$. The larger the value of θ implies that the E-Commerce operators are more critical about the honest consistency. We formally define a reputable seller and an average seller as follows.

Definition 2.1. *A seller is labeled as reputable if and only if the following two conditions are met*

C1: $r \geq r_h$ and,

C2: $n^+/(n^+ + n^- + n^0) \geq \theta$.

Otherwise, a seller is labeled as an average seller.

Hence, the reputation threshold r_h and consistency threshold θ quantify how difficult is it to earn a reputable label. The larger the r_h and θ , the more difficult it is to earn a reputable label.

A buyer estimates the product quality referring to the advertised quality Q_a and the reputation profile of a seller. More concretely, if a seller's reputation profile indicates that this seller is reputable, then a buyer believes that this seller advertises honestly. This buyer therefore estimates the product quality as the advertised quality, i.e., $Q_e = Q_a$. On the contrary, if the reputation profile indicates that a seller is average, a buyer believes that this seller is likely to overstate the product quality. Hence the estimated quality is lower than the advertised quality, i.e., $Q_e = \beta Q_a$, where $\beta \in [0, 1]$ denotes the discounting factor. The smaller the value of β implies that buyers are less willing to trust an average seller. We have

$$Q_e = \begin{cases} Q_a, & \text{if } r \geq r_h \text{ and } n^+/(n^+ + n^- + n^0) \geq \theta, \\ \beta Q_a, & \text{otherwise.} \end{cases}$$

A buyer makes the purchasing decision based on the estimated quality Q_e and the product price p . More concretely, the probability that a buyer buys a product increases in Q_e and decreases in p . Formally, we have

$$\Pr[\text{adopts a product}] \triangleq \mathcal{P}(Q_e, p), \quad (3)$$

where the function \mathcal{P} increases in Q_e and decreases in p .

In the following section, we evaluate the effectiveness of the baseline reputation system in E-commerce applications. Our goal is to identify key factors that influence the effectiveness of this reputation system and if possible, improve it.

3 Problems Formulation

We propose four performance measures to quantify the effectiveness of the baseline reputation system mentioned in Section 2. These measures are: (1) ramp up time T_r , (2) new seller drop out probability P_d , (3) long term expected profit gains for a seller G_s , and (4) average per seller transaction gains for the E-commerce system operator G_e . We also present our problem formulations and our objective is to identify key factors which can influence these measures. Lastly, we raise an interesting question of whether there are other mechanisms which can reduce the ramp up time and the new seller drop out probability, and improve the long term expected profit gains and average per seller transaction gains.

3.1 Ramp Up Time

Sellers and E-commerce system operators are interested in the minimum time that a new seller must spend to collect enough credits, i.e., positive feedbacks from buyers, so that the seller can be classified as reputable. For one thing, a reputable seller can attract more buyers which may result in more transactions, and higher transaction volume implies higher transaction gains to the E-commerce operator. We next formally define the ramp up process and the ramp up condition.

Definition 3.1. *A new seller's reputation is $r=0$. He needs to collect enough credits, i.e., positive feedbacks from buyers, so that his reputation r can increase to at least r_h . The process of increasing his reputation to r_h is called the ramp up process. Furthermore, when $r \geq r_h$, then we say that the ramp up condition is satisfied.*

Recall that $r(\tau)$ denotes the reputation score of a seller at time slot τ . We formally define the ramp up time as follows.

Definition 3.2. *Ramp up time is the minimum time that a seller must spend to accumulate a reputation score of r_h . Let T_r denote the ramp up time, we have*

$$T_r \triangleq d \cdot \arg \min_{\tau} \{r(\tau) \geq r_h\}. \quad (4)$$

The ramp up time quantifies how long it will take to collect enough credits from buyers. It is critical to the profit gains for a seller. To see this, we next quantify how the ramp up time can affect the transaction's arrival rate.

A seller can attract more buyers when he satisfies the ramp up condition because his online store will receive higher click rate by buyers, therefore increasing his profit gains. Let λ_1 (λ_2) be the buyer's arrival rate before (after) a seller satisfies the ramp up condition. We assume that the buyer's arrival process, either before or after a seller satisfies the ramp up condition, follows a Poisson counting process with parameter

λ_1 (before ramping up) and λ_2 (after ramping up) respectively, where $\lambda_1 < \lambda_2$ to signify that a ramped up seller can attract more buyers. Recall that in Equation (3) we express the probability that a buyer adopts a product as $\mathcal{P}(Q_e, p)$. If a buyer adopts a product, we say a seller obtains a transaction. Based on the Poisson property, it is easy to see that the transaction's arrival process is also a Poisson counting process. Let $\lambda_T(\tau)$ be the transaction's arrival rate at time slot τ . Let $\mathcal{P}(Q_e(\tau), p)$ be the probability that a buyer adopts a product at time slot τ , where $Q_e(\tau)$ denotes the estimated quality at time slot τ . We can express the transaction's arrival rate as

$$\lambda_T(\tau) = \begin{cases} \lambda_1 \mathcal{P}(Q_e(\tau), p), & \text{if } r(\tau) < r_h, \\ \lambda_2 \mathcal{P}(Q_e(\tau), p), & \text{if } r(\tau) \geq r_h, \end{cases} \quad (5)$$

Equation (5) serves as an important building block for us to explore the key factors which influence the ramp up time T_r . Let us formulate our first problem.

Problem 1: *Identify key factors which influence the ramp up time T_r , and design a mechanism which can take advantage of these factors to reduce T_r .*

3.2 New Seller Drop Out Probability

In real-world E-commerce systems, a new seller may drop out, or move to another E-commerce system, if he does not collect enough credits (i.e., ramp up) within certain time because he cannot obtain enough transactions. For example, a new seller in eBay may drop out if he does not ramp up in one year. This is because a new seller starts an online store with certain budgets and there are costs associated with maintaining this online business. Let $T_w > 0$ denote the maximum time that a new seller is willing to wait to get ramped up. In other words, if the ramp up time is longer than T_w , a new seller will quit or drop out from that E-commerce system. We assume $T_w/d \in \mathbb{N}$ to accommodate the delay (d) in reputation update.

Definition 3.3. *A new seller drops out, if and only if $T_r > T_w$.*

Sellers and the E-commerce operator are interested in this new seller drop out probability. Let

$$P_d \triangleq \Pr[T_r > T_w] \quad (6)$$

denote the probability that a new seller drops out. The smaller the value of P_d implies that sellers are more likely to continue his online business in the E-commerce system. This is an important measure since a small P_d can attract more new sellers to join the E-commerce system, which will result in higher transaction gains for the E-commerce operator. On the other hand, a large P_d discourages new sellers to participate and can reduce the transaction gains for the E-commerce operator. We therefore consider the second problem.

Problem 2: *Identify key factors which influence the new seller drop out probability P_d , and design a mechanism which can take advantage of these factors to reduce P_d .*

3.3 Long Term Profit Gains and Transaction Gains

The profit gain (transaction gain) is critical to sellers (E-commerce system operators). We focus on the scenario that sellers are long lived and they aim to maximize their long term profit gains. Recall that u , the unit profit of selling one product, is expressed in Equation (1). Also recall that $N(\tau)$ denotes the number of products sold in the time slot τ . We emphasize that $N(\tau)$ is a random variable and follows a Poisson distribution with parameter $\lambda_T(\tau)d$, where $\lambda_T(\tau)$ is derived in Equation (5). A seller earns a profit of $uN(\tau)$ in the time slot τ . We consider a discounted long term profit gain with a discounting factor of $\delta \in (0, 1]$. Let G_s denote the long term expected profit gains for a seller. We express it as

$$G_s \triangleq E \left[\sum_{\tau=0}^{\infty} \delta^{\tau} u N(\tau) \right]. \quad (7)$$

Note that when a seller earns a profit u , he also contributes a transaction fee $T = \alpha p$ to the E-commerce operator. Let G_e denote the average per seller transaction gains that a seller pays to the E-commerce operator. We can express it as

$$G_e \triangleq E \left[\sum_{\tau=0}^{\infty} \delta^{\tau} T N(\tau) \right] = E \left[\sum_{\tau=0}^{\infty} \delta^{\tau} \alpha p N(\tau) \right] = \frac{\alpha p}{u} G_s. \quad (8)$$

Note that G_s is important to a seller while G_e is important to the E-commerce operator. We consider the following problem.

Problem 3: Identify key factors which influence the profit gains G_s and average per seller transaction gains G_e , and design a mechanism to use these factors to improve G_e and G_s .

We next derive $E[T_r]$, P_d , G_s , and G_e . Through this analysis, we identify key factors which influence the above mentioned performance measures. These insights will serve as important building blocks for us to design a mechanism.

4 Analyzing the Baseline Reputation System

We derive analytical expressions for the expected ramp up time ($E[T_r]$), the new seller drop out probability (P_d), the long term expected profit gains (G_e) and the average per seller transaction gains (G_e). Through this we identify that the reputation threshold (r_h), as well as the probability that a buyer buys a product from an “average labeled” seller (P_{ab}) are two critical factors which influence T_r , P_d , G_s and G_e . Our results indicates that the baseline reputation mechanism described in Section 2, suffers from long ramp up time, high new seller drop out probability, and small long term profit gains or transaction gains. These insights show that one need to have a new mechanism to reduce T_r , P_d , and to improve G_s and G_e . We will present this new mechanism in Section 5.

4.1 Deriving the Expected Ramp Up Time $E[T_r]$

Let us derive the analytical expression for the expected ramp up time $E[T_r]$. This measure quantifies on average, how long it will take to ramp up a new seller under the baseline reputation mechanism mentioned in Section 2. We consider the scenario that buyers advertise the product quality honestly, i.e., $Q_a = Q_i$. As to how an eBay-like reputation mechanism can guarantee rational sellers to advertise honestly, one can refer to [8]. We like to point out that new sellers can achieve the *lowest ramp up time* by advertising honestly ($Q_a = Q_i$). Hence, the assumption that $Q_a = Q_i$ can be viewed as deriving the best case of T_r for the baseline reputation system. Let us first define the following notations.

Definition 4.1. Let $P_{ba} \triangleq \mathcal{P}(\beta Q_i, p)$ and $P_{br} \triangleq \mathcal{P}(Q_i, p)$ denote the probability that a buyer buys a product from an “average labeled” seller and a “reputable” seller respectively.

In the following theorem, we state the expected ramp up time.

Theorem 4.1. The expected ramp up time is

$$E[T_r] = d \sum_{\tau=1}^{\infty} \left(1 - \sum_{k=r_h}^{\infty} e^{-\lambda_1 P_{ba}(\tau-1)d} \frac{(\lambda_1 P_{ba}(\tau-1)d)^k}{k!} \right).$$

The $E[T_r]$ increases in the reputation threshold r_h , and decreases in the transaction’s arrival rate $\lambda_1 P_{ba}$.

Proof: Please refer to the appendix for derivation. ■

Remark: Theorem 4.1 states that a new seller is more difficult to get ramped up if the E-Commerce operator sets a high reputation threshold r_h , or the transaction’s arrival rate to an “average labeled” seller ($\lambda_1 P_{ba}$) is low.

Table 2 presents numerical examples on the expected ramp up time $E[T_r]$, where we fix $P_{ba} = 0.02$, i.e., buyers buy products from an “average labeled” seller with probability 0.02, and fix $d = 3$, i.e., it takes three days to ship a product to a buyer (or for the E-commerce operator to update sellers’ reputation). We vary the buyer’s arrival rate λ_1 from 5 to 25, i.e., on average each day an “average labeled” seller attracts 5 to 25 buyers to visit his online store. We vary the reputation threshold (r_h) from 100 to 200. When $r_h = 200$, as λ_1 increases from 5 to 25, the expected ramp up time ($E[T_r]$) drops from 2001.7 to 401.6 days, a deduction ratio of 80%. When the buyer’s arrival rate is low, say $\lambda_1 = 5$, as the reputation threshold r_h drops from 200 to 100, the expected ramp up time $E[T_r]$ drops from 2001.7 to 1001.4 days, a reduction ratio of 50%. These results indicates that the expected ramp up time ($E[T_r]$) is large in general. Namely, it is difficult for new sellers to quickly get ramped up under the baseline reputation system. We next explore the new seller drop out probability.

4.2 Deriving the New Seller Drop Out Probability P_d

We now derive the analytical expression for P_d . This probability quantifies how difficult it is for a new seller to *survive* in the E-commerce system. Note that P_d is also crucial for new sellers to decide whether or not to open an online store in an E-commerce system. Namely, a low drop out probability P_d is attractive to new sellers, while a high P_d discourages new sellers to join.

λ_1	5	10	15	20	25
$E[T_r]$ ($r_h = 200$)	2001.7	1001.4	668.2	501.5	401.6
$E[T_r]$ ($r_h = 150$)	1501.4	751.5	501.5	376.5	301.5
$E[T_r]$ ($r_h = 100$)	1001.4	501.5	334.8	251.5	201.5

Table 2: Expected ramp up time $E[T_r]$ in days ($P_{ba} = 0.02, d=3$).

Theorem 4.2. *The new seller drop out probability is*

$$P_d = \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w} \frac{(\lambda_1 P_{ba} T_w)^k}{k!}.$$

The P_d decreases in $\lambda_1 P_{ba}$, T_w and increases in r_h .

Proof: Please refer to the appendix for derivation. \blacksquare

Remark: Theorem 4.2 states that a new seller can reduces the drop out probability by extending his ramp up deadline line (T_w), and a new seller is more likely to drop out if the reputation threshold (r_h) increases or the transaction's arrival rate to an “average labeled” seller ($\lambda_1 P_{ba}$) decreases.

Table 3 presents numerical examples on the new seller drop out probability P_d , where we set $\lambda_1 = 20$, i.e., on average, each day an “average labeled” seller attracts 20 buyers to visit his store, $d=3$, and $T_w=180$, i.e., sellers drops out if they do not ramp up in 180 days. We vary P_{ba} , the probability that a buyer buys products from an “average labeled” seller, from 0.01 to 0.05, and vary the reputation threshold r_h from 100 to 200. Consider $r_h = 200$. As P_{ba} increases from 0.01 to 0.05, the new seller drop out probability P_d decreases from 1 to 0.92514. This implies a very high drop out probability. Consider $P_{ba}=0.03$. As the reputation threshold r_h drops from 200 to 100, we see that P_d drops from 1 to 0.20819, a reduction ratio of around 80%. It is interesting to observe that when the P_{ba} is small, the new seller drop out probability is quite high. In fact when $P_{ba}=0.01$, P_d is very close to 1. In other words, if buyers are less willing to buy from “average labeled” sellers, new sellers will be more likely to drop out. We next explore key factors which influence long term expected profit gains and average per seller transaction gains.

P_{ba}	0.01	0.02	0.03	0.04	0.05
P_d ($r_h = 200$)	1.00000	1.00000	1.00000	0.99999	0.92514
P_d ($r_h = 150$)	1.00000	1.00000	0.99992	0.68056	0.00991
P_d ($r_h = 100$)	1.00000	0.99897	0.20819	0.00005	0.00000

Table 3: New seller drop out probability P_d ($\lambda_1 = 20, T_w=180, d=3$).

4.3 Deriving the Long Term Profit Gains G_s and G_e

Let us now derive analytical expressions for the long term expected profit gains G_s and the average per seller transaction gains G_e respectively. They are important measures because a large G_s is attractive to new sellers and a small G_s discourages new sellers to join the E-commerce system, while the average per seller transaction gains G_e is crucial to the E-commerce system operator.

Theorem 4.3. *The long term expected profit gains for a new seller can be expressed as*

$$\begin{aligned} G_s = & \sum_{k=0}^{r_h-1} \frac{(\lambda_1 P_{ba} T_w)^k}{(k)!} e^{-\lambda_1 P_{ba} T_w} \int_{0 < t_1 < \dots < t_k < T_w} \frac{k!}{T_w^k} \sum_{j=1}^k u \\ & \delta^{\lceil t_j/d \rceil} dt_1 \dots dt_k + \int_0^{T_w} dt_{r_h} \int_{0 < t_1 < \dots < t_{r_h-1} < t_{r_h}} \lambda_1 P_{ba} e^{-\lambda_1 P_{ba} t_{r_h}} \\ & \frac{(\lambda_1 P_{ba} t_{r_h})^{r_h-1}}{(r_h-1)!} \frac{(r_h-1)!}{(t_{r_h})^{r_h-1}} u \left(\sum_{j=1}^{r_h} \delta^{\lceil t_j/d \rceil} + \lambda_2 P_{br} d \frac{\delta^{\lceil t_{r_h}/d \rceil} + 1}{1-\delta} \right. \\ & \left. + \delta^{\lceil t_{r_h}/d \rceil} \lambda_1 P_{ba} (d^{\lceil t_{r_h}/d \rceil} - t_{r_h}) \right) dt_1 \dots dt_{r_h-1}. \end{aligned}$$

Furthermore, $G_e = \frac{\alpha p}{u} G_s$.

Proof: Please refer to the appendix for derivation. ■

Remark: Theorem 4.3 quantifies the impact of various factors on the long term expected profit gains (G_s) and average per seller transaction gains (G_e), e.g., the reputation threshold r_h , the buyer's arrival rate λ_1, λ_2 , etc. However the analytical expressions are in complicated forms. It is hard to draw some insights on how critical are these factors in influencing G_s and G_e by directly examining these expressions. We next present some numerical examples to illustrate their impact.

Table 4 presents numerical examples on the long term expected profit gains (G_s) and the average per seller transaction gains (G_e), where we set $\lambda_1 = 20$, $T = \alpha p = 0.1$, $\lambda_2 = 50$, $u = 1$, $T_w = 180$, $d = 3$, $P_{br} = 0.1$ (i.e., buyers buy products from a reputable seller with probability 0.1). We vary P_{ba} , the probability that a buyer purchases products from an “average labeled” seller, and the ramp up threshold r_h to examine their impact on G_s and G_e . Consider $r_h = 200$. As P_{ba} increases from 0.01 to 0.05, G_s improves from 26.833 to 198.059, an improvement ratio of 6.38 time. Similarly, the average per seller transaction gains G_e is also improved by 6.38 times. This implies that P_{ba} is critical to both sellers' profit gains and the E-commerce system operator's transaction gains. Consider $P_{ba} = 0.05$. As r_h drops from 200 to 100, G_s improves from 198.059 to 1142.670, an improvement ratio of 4.77. This improvement ratio also holds for the average per seller transaction gains G_e . It is interesting to observe that when P_{ba} is small, both G_s and G_e are quite small. In fact when $P_{ba} = 0.01$, the G_s is around 26.833 and G_e is around 2.6833. Namely, if buyers are less willing to buy from “average labeled” sellers, sellers (E-commerce operators) will have low long term profit gains (average per seller transaction gains).

P_{ba}	0.01	0.02	0.03	0.04	0.05
G_s ($r_h = 200$)	26.833	53.342	80.705	107.312	198.059
G_s ($r_h = 150$)	26.980	53.594	80.812	369.951	1006.017
G_s ($r_h = 100$)	26.941	54.433	760.511	1054.507	1142.670
G_e ($r_h = 200$)	2.6833	5.3342	8.0705	10.7312	19.8059
G_e ($r_h = 150$)	2.6980	5.3594	8.0812	36.9951	100.6017
G_e ($r_h = 100$)	2.6941	5.4433	76.0511	105.4507	114.2670

Table 4: Long term expected profit gains G_s and average per seller transaction gains G_e ($\lambda_1 = 20, \lambda_2 = 50, u = 1, T = \alpha p = 0.1, \delta = 0.99, T_w = 180, P_{br} = 0.1, d = 3$).

Summary: The reputation threshold r_h and P_{ba} are critical to the ramp up time, the new seller drop out probability, the long term profit gains and the average per seller transaction gains. The baseline (or eBay-like) reputation system presented in Section 2 suffers from long ramp up time, high seller drop out probability, small long term profit gains and small average per seller transaction gains. Hence, it is important to ask whether we can design a new mechanism that an E-commerce system can use to improve all the performance measures $E[T_r], P_d, G_s$ and G_e . We next explore this interesting question.

5 Insurance Mechanism

In the previous section, we showed that the baseline reputation system is not efficient. Here, we present a new approach which we call the “*insurance mechanism*” to reduce both the expected ramp up time ($E[T_r]$) and new seller drop out probability (P_d), and to improve the long term expected profit gains (G_s) and the average per seller transaction gains (G_e). We also quantify the impact of our insurance mechanism on $E[T_r], P_d, G_s$ and G_e . We show that our insurance mechanism can reduce the ramp up time by around 87.2%, and guarantee new sellers ramp up before the deadline T_w with a high probability (close to 1.0). It also increases the long term profit gains and average per seller transaction gains by at least 95.3%.

5.1 Insurance Mechanism Design

The objective of our insurance mechanism is to help new sellers ramp up quickly. Reducing ramp up time brings the benefit of reducing new seller drop out probability and improving long term expected profit gains and average per seller transaction gains (G_s and G_e). Our insurance mechanism consists of an *insurance protocol* and a *transaction mechanism*.

We first describe the *insurance protocol*. The E-commerce system operator provides an insurance service to new sellers. Each insurance has a price of $C_I > 0$, a duration time of $T_d > 0$, and a clearing time of $T_c > 0$. The insurance clearing time takes effect when an insurance expires. To buy an insurance, a seller must provide the E-commerce operator an insurance deposit of D_I . Hence, the total payment by the new seller

to the E-commerce system operator is $C_I + D_I$. We refer to this insurance as the (C_I, T_d, T_c, D_I) -insurance. Only new sellers can subscribe to this insurance. If a seller subscribes an insurance, the E-commerce system operator issues an insurance certificate to him, and this certificate is known to the public (i.e., all buyers and sellers). This certificate only takes effect within the insurance duration time T_d . The E-commerce system operator treats a seller with an insurance certificate as trustworthy. To guarantee that such sellers will advertise their product quality honestly, the E-commerce system operator requires such sellers obey the following *transaction mechanism*.

We now describe the *transaction mechanism*. Only sellers with an insurance certificate must obey this transaction mechanism. Let us focus on a seller with an insurance certificate. When ordering a product from this seller, a buyer sends his payment p to the E-commerce system operator. After receiving the product, if this buyer express a positive feedback, then this transaction completes, i.e., the E-commerce operator forwards the payment $(1 - \alpha)p$ to the seller and charges a transaction fee of αp . This transaction also completes if this buyer expresses a neutral feedback. A neutral feedback means that a seller slightly overstated his product quality, i.e. $Q_i < Q_a < Q_i + \gamma$. To avoid such overstating, the E-commerce company revokes a seller's insurance certificate once the fraction of positive feedbacks falls below the consistency factor (θ) , i.e., $n^+/(n^+ + n^0 + n^-) < \theta$. A negative feedback results in the transaction being revoked. More concretely, the E-commerce operator gives the payment p back to the buyer and does not charge any transaction fee from the seller (provided that it is within the duration time T_d , or the clearing time T_c). The buyer needs to ship the product back to the seller but the buyer does not need to pay for the shipment cost C_S , for it will be deducted from a seller's insurance deposit D_I . If the insurance deposit is not enough to cover C_S , the E-commerce operator makes a supplemental payment. To avoid this undesirable outcome, the E-commerce company revokes a seller's insurance certificate, once a seller's deposit reaches a threshold $\widehat{D}_I < D_I$. The insurance clearing time takes effect when an insurance is invoked. At the end of the clearing time, the E-commerce company returns the remaining deposit of D_I (if it is not deducted to zero) back to the seller.

Remark. Sellers may collude with buyers to inflate their reputation by fake transactions [10]. One way to avoid such collusion is by increasing the transaction fee such as [2]. The shipment cost may exceed D_I due to a large number of products to be returned. This can be avoided with high probability by setting a large \widehat{D}_I (Theorem 5.2). We also derive the minimum clearing time (T_c) to guarantee that a seller with an insurance certificate needs to obey the transaction mechanism (Theorem 5.2).

5.2 Analyzing the Insurance Mechanism

We first show that buyers treat a seller having an insurance certificate as trustworthy. Through this, we derive the transaction rate that a seller with an insurance certificate can attract. We then derive the improved $E[T_r], P_d, G_s$ and G_e .

Buyers treat sellers having an insurance certificate as trustworthy. This is an important property of our insurance mechanism because it influences the probability that a buyer adopts a product from a seller. Suppose in time slot τ , a seller has an insurance certificate. If this seller advertise honestly $Q_a = Q_i$, then the buyer who buys a product from this seller will be satisfied (express positive feedback rating). In this case, the payment from the buyer will be forwarded to the seller. Hence this seller earns a profit of u . If this seller overstates his product quality beyond the lenient factor (γ) , i.e., $Q_a > Q_i + \gamma$. Then according to our insurance mechanism, the payment by the buyer will be returned back to the buyer. The seller needs to pay a shipment cost of C_S to ship back the product and C_S will be deducted from his insurance deposit D_I . Hence, if a seller overstates the product quality beyond the lenient factor, he will lose a total shipment cost of at least $\min\{\widehat{D}_I, C_S N(\tau)\}$ in time slot τ , where $N(\tau)$ denotes the number of product selling. A seller with an insurance certificate must obey the same consistency factor (θ) as reputable sellers in being honest, i.e., $n^+/(n^+ + n^0 + n^-) \geq \theta$, because if not his insurance certificate will be revoked by the E-commerce operator. Given these properties, buyers trust a seller with an insurance certificate. Recall that the E-commerce operator also trusts a seller with an insurance certificate. Therefore, an insured seller can attract transactions with an arrival rate being equivalent to those reputable sellers. Let λ_T^I denote the transaction's arrival rate to a seller with an insurance certificate. We have

$$\lambda_T^I = \lambda_2 P_{br}. \quad (9)$$

We now quantify the impact of our insurance mechanism on the four performance measures. Let $T_r^I, P_d^I, G_s^I, G_e^I$ denote the ramp up time, the new seller drop out probability, the long term profit gains and the average per seller transaction gains respectively, when a new seller subscribes our insurance.

Theorem 5.1. *Suppose a new seller subscribes to our proposed insurance mechanism. We express the*

expected ramp up time and new seller drop out probability as

$$E[T_r] = d \sum_{\tau=1}^{\infty} \left(1 - \sum_{k=r_h}^{\infty} e^{-\sum_{\ell=0}^{\tau-2} \tilde{\lambda}_T(\ell)} \frac{(\sum_{\ell=0}^{\tau-2} \tilde{\lambda}_T(\ell))^k}{k!} \right),$$

$$P_d = \sum_{k=0}^{r_h-1} e^{-\sum_{\ell=0}^{T_w/d-1} \tilde{\lambda}_T(\ell)} \frac{(\sum_{\ell=0}^{T_w/d-1} \tilde{\lambda}_T(\ell))^k}{k!},$$

where $\tilde{\lambda}_T(\ell) = \lambda_2 P_{br} d$ for all $\ell = 0, 1, \dots, \lfloor T_d/d \rfloor - 1$, and $\tilde{\lambda}_T(\lfloor T_d/d \rfloor) = \lambda_2 P_{br} (T_d - d \lfloor T_d/d \rfloor) + \lambda_1 P_{ba} (d \lfloor T_d/d \rfloor + d - T_d)$, and $\tilde{\lambda}_T(\ell) = \lambda_1 P_{ba} d$ for all $\ell = \lfloor T_d/d \rfloor + 1, \dots, \infty$. The long term expected profit gains for an insured seller is:

$$G_s^I = \sum_{k=0}^{r_h-1} \frac{(\lambda_2 P_{br} \min\{T_d, T_w\})^k}{k!} e^{-\lambda_2 P_{br} \min\{T_d, T_w\}}$$

$$\int_{0 < t_1 < \dots < t_k < \min\{T_d, T_w\}} \int \frac{k! dt_1 \dots dt_k}{(\min\{T_d, T_w\})^k} \left(\mathbf{I}_{\{T_d \geq T_w\}} \sum_{j=1}^k u \delta^{\lceil t_j/d \rceil} \right.$$

$$+ \mathbf{I}_{\{T_d < T_w\}} \sum_{i=0}^{r_h-1-k} \frac{(\lambda_1 P_{ba} (T_w - T_d))^i}{i!} e^{-\lambda_1 P_{ba} (T_w - T_d)}$$

$$\left. \int_{T_d < t_{k+1} < \dots < t_{k+i} < T_w} \int \frac{i!}{(T_w - T_d)^i} dt_{k+1} \dots t_{k+i} \sum_{j=1}^{k+i} u \delta^{\lceil t_j/d \rceil} \right)$$

$$+ \left(\mathbf{I}_{\{T_d \geq T_w\}} \int_0^{T_w} \lambda_2 P_{br} e^{-\lambda_2 P_{br} t_{r_h}} \frac{(\lambda_2 P_{br} t_{r_h})^{r_h-1}}{(r_h-1)!} dt_{r_h} \right.$$

$$\left. \int_{0 < t_1 < \dots < t_{r_h-1} < t_{r_h}} \int \frac{(r_h-1)!}{(t_{r_h})^{r_h-1}} dt_1 \dots dt_{r_h-1} + \mathbf{I}_{\{T_d < T_w\}} \sum_{k=0}^{r_h} \right.$$

$$\frac{(\lambda_2 P_{br} T_d)^k}{k!} e^{-\lambda_2 P_{br} T_d} \int_{0 < t_1 < \dots < t_k < T_d} \int \frac{k!}{T_d^k} dt_1 \dots dt_k$$

$$\left. \int_{T_d}^{T_w} \lambda_1 P_{ba} e^{-\lambda_1 P_{ba} (t_{r_h} - T_d)} \frac{(\lambda_1 P_{ba} (t_{r_h} - T_d))^{r_h-k-1}}{(r_h-k-1)!} dt_{r_h} \right)$$

$$\int_{T_d < t_{k+1} < \dots < t_{r_h-1} < t_{r_h}} \int \frac{(r_h-k-1)! dt_{k+1} \dots dt_{r_h-1}}{(t_{r_h} - T_d)^{r_h-k-1}} u \left(\lambda_2 d P_{br} \right.$$

$$\left. \frac{\delta^{\lceil t_{r_h}/d \rceil} + 1}{1-\delta} + \sum_{j=1}^{r_h} \delta^{\lceil t_j/d \rceil} + \delta^{\lceil t_{r_h}/d \rceil} \lambda_2 P_{br} (d \lceil t_{r_h}/d \rceil - t_{r_h}) \right)$$

Furthermore, $G_e^I = \frac{\alpha p}{u} G_s^I$.

Proof: Please refer to the appendix for derivation. ■

Remark: Theorem 5.1 quantifies the impact of our insurance mechanism on the four important performance measures. Before we talk more about how to select the insurance price C_I and deposit D_I , let us illustrate the effectiveness of our insurance mechanism using some numerical examples.

Table 5 presents numerical examples on $E[T_r^I]$, P_d^I , G_s^I and G_e^I . We use the following setting: $\lambda_1 = 20$, $\lambda_2 = 50$, $u = 1$, $T_w = 180$, $d = 3$, $P_{br} = 0.1$, $P_{ba} = 0.03$, $C_I = 100$, $D_I = 100$, $\hat{D}_I = 50$, $C_S = 0.5$, $T = 0.1$, $T_d = 100$, $T_c = 3$, $\delta = 0.99$. We also presents numerical examples on $E[T_r]$, P_d , G_s and G_e for comparison studies. When $r_h = 100$, we have $E[T_r] = 168.1$ and $E[T_r^I] = 21.5$. In other words, our insurance mechanism reduces the expected ramp up time from 168.1 days to only 21.5 days, or over 87.2% reduction. It is interesting to observe that our incentive mechanism reduces the new seller drop out probability from $P_d = 0.20819$ to $P_d^I = 0$. Namely, our insurance mechanism can guarantee that new sellers ramps up before the deadline line T_w with a high probability (very close to 1.0). In addition, our insurance mechanism improves long term expected profit gains from $G_s = 760.51$ to $G_s^I = 1485.04$, a 95.3% improvement. This improvement ratio also holds for average per seller transaction gains. As r_h increases from 100 to 200, the improvement on the $E[T_r]$, P_d , G_s , G_e , becomes more significant. We next state the appropriate values for C_I , D_I , \hat{D}_I and T_c in the following theorem.

	$r_h = 100$	$r_h = 150$	$r_h = 200$
$(E[T_r], E[T_r^I])$	(168.1, 21.5)	(251.6, 31.5)	(334.9, 41.5)
(P_d, P_d^I)	(0.20819, 0)	(0.99992, 0)	(1.0, 0)
(G_s, G_s^I)	(760.51, 1485.04)	(80.81, 1485.03)	(80.71, 1485.01)
(G_e, G_e^I)	(76.051, 148.504)	(8.081, 148.503)	(8.071, 148.501)

Table 5: Impact of our insurance on $E[T_r]$, P_d , G_s and G_e .

Theorem 5.2. *An upper bound for the insurance price C_I is $C_I < G_s^I - G_s$. If D_I and \hat{D}_I satisfies*

$$D_I > \hat{D}_I \geq C_S \max\{\ln \epsilon^{-1} - \lambda_2 P_{br} T_d, e^2 \lambda_2 P_{br} T_d\},$$

then $\Pr[\text{shipment cost exceeds } D_I] \leq \epsilon$. If $T_c \geq d$, then all products sold by a seller with an insurance certificate can be guaranteed to obey the insurance mechanism.

Proof: Please refer to the appendix for derivation. ■

Remark: The insurance price should be lower than $G_s^I - G_s$. The clearing time should be larger or equal to d . To guarantee that the insurance deposit covers the shipment cost for returning products with high probability, D_I and \hat{D}_I need to be no less than $C_S \max\{\ln \epsilon^{-1} - \lambda_2 P_{br} T_d, e^2 \lambda_2 P_{br} T_d\}$.

6 Related Work

Research on reputation systems [19] for internet services has been quite active. Many aspects of reputation systems have been studied, i.e., reputation metric formulation and calculation [11, 15, 20], attacks and defense techniques for reputation systems [5, 10, 24, 23], and effectiveness of reputation systems [7]. A survey can be found in [12].

Theoretical aspects of reputation system have been studied extensively. First, many works studied reputation metric formulation and calculation. Two most representative reputation calculating models are the eBay-like reputation model [11] and the transitive trust based model [5]. The eBay-like reputation system is a typical example of reputation model which computes the reputation score by summarizing explicit human feedbacks (or ratings) [3, 11, 21, 25]. The transitive trust based model [5, 6, 20, 15, 24] assumes that if user A trusts user B and user B trusts user C , then user A trusts user C . More precisely, each user is represented by a node in a graph, and the weighted directed link from A to B quantifies the degree that user A trusts user B . For this model, many algorithms were developed to compute an overall reputation score for each user [5, 6, 20, 15, 24]. These works provided theoretical foundations for reputation computing. Second, many works explored attack and defense techniques for reputation systems. One type of potential attacks is that users may not give honest feedbacks. Peer-prediction method based mechanisms were proposed to elicit honest feedbacks [13, 14, 17]. Another type of potential attacks is reputation inflation, or self-promotion. Many works have been done to address this issue [5, 10, 24, 23]. A survey on attack and defense techniques for reputation systems can be found in [10]. The main difference between our work and theirs is that we propose a new mechanism to improve eBay system.

The most closely related works are [2, 7, 8, 16], which studied the eBay reputation mechanism. Authors in [2] derived the minimum transaction fee to avoid ballot stuffing (i.e., fake positive feedbacks). Authors in [7] proposed an algorithm based on buyer friendship relationship to filter out unfair ratings. In [7], authors explored the impact of buyers biases' (i.e., leniency or criticality) in express feedback ratings on sellers in advertising product quality. The impact of negative feedbacks on buyers in expressing feedback ratings was studied in [16]. The difference between our work and theirs is that we propose a new mechanism to improve eBay system.

7 Conclusion

This paper presents an Insurance mechanism to improve eBay-like reputation mechanisms. We proposed four performance measures to analyze eBay reputation system: (1) new seller ramp up time, (2) new seller drop out probability, (3) long term profit gains for sellers and (4) average per seller transaction gains for an E-commerce operator. We developed a stochastic model to identify key factors which influence the above four measures. We proposed an insurance mechanism to improve the above four measures. We show that our insurance mechanism can reduce the ramp up time by around 87.2%, and guarantee new sellers ramp up

before the deadline T_w with a high probability (close to 1.0). It also increases the long term profit gains and average per seller transaction gains by at least 95.3%.

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Appendix

We first state a lemma which will be used in our proof.

Lemma 7.1 ([4]). *Let $\{N'(t), t \geq 0\}$ denote a Poisson process with a rate parameter λ . Let t_k denote the arrival time of k -th event. Let $f(t_1, \dots, t_n | N'(t') = n)$ denote the conditional probability density function of t_1, \dots, t_n given $N'(t') = n$. Then we have $f(t_1, \dots, t_n | N'(t') = n) = n!/(t')^n$, where $0 < t_1 < \dots < t_n < t'$.*

Proof of Theorem 4.1: Note that each new seller advertise product quality honestly. In this scenario, each transaction results in one positive feedback. Recall our reputation updating rule specified in Equation (2), we have that the reputation score at time slot τ equals the number of transactions arriving within time slot 0 to time slot $\tau - 1$. Recall the definition of T_r in Equation (4), we have that $T_r/d \in \mathbb{N}$. With these observations and by some basic probability arguments, we have

$$\begin{aligned} E[T_r] &= \sum_{\tau=1}^{\infty} \tau d \Pr[T_r = \tau d] = d \sum_{\tau=1}^{\infty} \tau \Pr[T_r/d = \tau] \\ &= d \sum_{\tau=1}^{\infty} \Pr[T_r/d \geq \tau] = d \sum_{\tau=1}^{\infty} (1 - \Pr[T_r/d \leq (\tau - 1)]) \\ &= d \sum_{\tau=1}^{\infty} (1 - \Pr[r(\tau - 1) \geq r_h]) = d \sum_{\tau=1}^{\infty} \left(1 - \sum_{\ell=0}^{\tau-2} N(\ell) \geq r_h \right). \end{aligned}$$

Note that $\sum_{\ell=0}^{\tau-2} N(\ell)$ is a random variable which follows a Poisson distribution with parameter $\lambda_1 P_{ba}(\tau - 1)d$. We have

$$E[T_r] = d \sum_{\tau=1}^{\infty} \left(1 - \sum_{k=r_h}^{\infty} e^{-\lambda_1 P_{ba}(\tau - 1)d} \frac{(\lambda_1 P_{ba}(\tau - 1)d)^k}{k!} \right).$$

Evaluating the first order derivative on $E[T_r]$ with respect to r_h and $\lambda_1 P_{ba}$ respectively, one can easily obtain the monotonous property of $E[T_r]$. \blacksquare

Proof of Theorem 4.2: Applying similar derivation as Theorem 4.1, we have that the reputation score at time slot τ equals the number of transactions arriving within time slot 0 to $\tau - 1$. Note that $T_w/d \in \mathbb{N}$. Recall the definition in Equation (4) we have that $T_r > T_w$ if and only if $r(T_w/d) < r_h$. Using some basic probability arguments, we have

$$P_d = \Pr[r(T_w/d) < r_h] = \Pr \left[\sum_{\tau=0}^{T_w/d-1} N(\tau) < r_h \right].$$

Note that $\sum_{\tau=0}^{T_w/d-1} N(\tau)$ is a random variable which follows a Poisson distribution with parameter $\lambda_1 P_{ba} T_w$. We have

$$P_d = \sum_{k=0}^{r_h-1} \Pr \left[\sum_{\tau=0}^{T_w/d-1} N(\tau) = k \right] = \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w} \frac{(\lambda_1 P_{ba} T_w)^k}{k!}.$$

Evaluating the first order derivative on P_d with respect to r_h , T_w and $\lambda_1 P_{ba}$ respectively, one can easily obtain the monotonous property of P_d . \blacksquare

Proof of Theorem 4.3: Let $\widehat{G}_s = \sum_{\tau=0}^{\infty} \delta^{\tau} u N(\tau)$. Then \widehat{G}_s is a random variable and $G_s = E[\widehat{G}_s]$. Based on whether a seller ramps up or not, we divide $E[\widehat{G}_s]$ into two parts, i.e.,

$$\begin{aligned} E[\widehat{G}_s] &= \Pr[T_r > T_w] E[\widehat{G}_s | T_r > T_w] \\ &\quad + \Pr[T_r \leq T_w] E[\widehat{G}_s | T_r \leq T_w] \end{aligned}$$

We next derive the above two terms individually.

We first derive $\Pr[T_r > T_w]E[\hat{G}_s|T_r > T_w]$. Note that each new seller advertise product quality honestly. Hence each transaction earns one positive feedback. Recall that $T_w/d \in \mathbb{N}$. Note that at time slot T_w/d a seller drops out, i.e., there will be no transactions from time slot T_w/d . Let K denote the number of transactions arriving within time slot 0 to $T_w/d - 1$. Then K satisfies $0 \leq K \leq r_h - 1$ since we are given $T_r > T_w$. Note that $K = k$, where $k \leq r_h - 1$ implies that $T_r > T_w$ and K is of a value larger than $r_h - 1$ implies that $T_r \leq T_w$. Then we have

$$\begin{aligned} & \Pr[T_r > T_w]E[\hat{G}_s|T_r > T_w] \\ &= \sum_{k=0}^{r_h-1} \Pr[K = k, T_r > T_w]E[\hat{G}_s|T_r > T_w, K = k] \\ &= \sum_{k=0}^{r_h-1} \Pr[K = k]E[\hat{G}_s|K = k]. \end{aligned}$$

Observe that $K = \sum_{\tau=0}^{T_w/d-1} N(\tau)$ is a random variable which follows a Poisson distribution with parameter $\lambda_1 P_{ba} T_w$. We then have

$$\begin{aligned} & \Pr[T_r > T_w]E[\hat{G}_s|T_r > T_w] \\ &= \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w} \frac{(\lambda_1 P_{ba} T_w)^k}{k!} E[\hat{G}_s|K = k]. \end{aligned}$$

We next derive $E[\hat{G}_s|K = k]$. Let t_1, \dots, t_K denote the arrival time of transaction $1, \dots, K$. Then t_1, \dots, t_K satisfy $0 < t_1 < \dots < t_K < T_w$. Let $f(t_1, \dots, t_k|K = k)$ denote the probability density function of t_1, \dots, t_k given that $K = k$. By applying Lemma 7.1, we obtain that $f(t_1, \dots, t_k|K = k) = k!/(T_w)^k$. Then we have

$$\begin{aligned} & E[\hat{G}_s|K = k] \\ &= \int \dots \int_{0 < t_1 < \dots < t_k < T_w} f(t_1, \dots, t_k|K = k) E[\hat{G}_s|t_1, \dots, t_k] \\ & \quad dt_1 \dots dt_k \\ &= \int \dots \int_{0 < t_1 < \dots < t_k < T_w} \frac{k!}{T_w^k} \sum_{j=1}^k u \delta^{\lceil t_j/d \rceil} dt_1 \dots dt_k, \end{aligned}$$

where the first step follows that given t_1, \dots, t_k is equivalent to given $K = k$. The second step follows that the payment of transaction $1 \leq j \leq k$ is forwarded to the seller in time slot $\lceil t_j/d \rceil$ because of the shipment delay. Namely, in computing of the long term profit gains, the j -th transaction results in a discounted profit gain of $u \delta^{\lceil t_j/d \rceil}$. Hence $E[\hat{G}_s|t_1, \dots, t_k] = \sum_{j=1}^k u \delta^{\lceil t_j/d \rceil}$. Combining them all, we have

$$\begin{aligned} \Pr[T_r > T_w]E[\hat{G}_s|T_r > T_w] &= \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w} \frac{(\lambda_1 P_{ba} T_w)^k}{k!} \\ & \quad \int \dots \int_{0 < t_1 < \dots < t_k < T_w} \frac{k!}{T_w^k} \sum_{j=1}^k u \delta^{\lceil t_j/d \rceil} dt_1 \dots dt_k. \end{aligned}$$

We now derive $\Pr[T_r \leq T_w]E[\hat{G}_s|T_r \leq T_w]$. Let t_{r_h} denote the arrival time of the r_h -th transaction. Based on our reputation updating rule specified in Equation (2), we obtain that $t_{r_h} < T_w$ implies $T_r \leq T_w$. Note that t_{r_h} is a random variable. Let $f(t_{r_h})$ denote the probability density function of t_{r_h} . Observe that $\Pr[t_{r_h} \leq t] = \sum_{k=r_h}^{\infty} e^{-\lambda_1 P_{ba} t} (\lambda_1 P_{ba} t)^k / k!$. Performing the first order derivative on this term, we have $f(t_{r_h}) = \lambda_1 P_{ba} e^{-\lambda_1 P_{ba} t_{r_h}} (\lambda_1 P_{ba} t_{r_h})^{r_h-1} / (r_h - 1)!$. Then it follows that

$$\begin{aligned} \Pr[T_r \leq T_w]E[\hat{G}_s|T_r \leq T_w] &= \int_0^{T_w} f(t_{r_h}) E[\hat{G}_s|t_{r_h}] dt_{r_h} \\ &= \int_0^{T_w} \lambda_1 P_{ba} e^{-\lambda_1 P_{ba} t_{r_h}} \frac{(\lambda_1 P_{ba} t_{r_h})^{r_h-1}}{(r_h - 1)!} E[\hat{G}_s|t_{r_h}] dt_{r_h}. \end{aligned}$$

We next derive $E[\hat{G}_s|t_{r_h}]$. Let t_1, \dots, t_{r_h-1} denote the arrival time of the 1-st, ..., $(r_h - 1)$ -th transaction. Let $f(t_1, \dots, t_{r_h-1}|t_{r_h})$ denote the probability density function of t_1, \dots, t_{r_h-1} given t_{r_h} . Then applying

Lemma 7.1, we obtain that $f(t_1, \dots, t_{r_h-1} | t_{r_h}) = (r_h - 1)! / (t_{r_h})^{r_h-1}$. Then it follows that

$$\begin{aligned} E[\widehat{G}_s | t_{r_h}] &= \int_{0 < t_1 < \dots < t_{r_h-1} < t_{r_h}} \int f(t_1, \dots, t_{r_h-1} | t_{r_h}) \\ &\quad E[\widehat{G}_s | t_1, \dots, t_{r_h-1}, t_{r_h}] dt_1 \dots dt_{r_h-1} \\ &= \int_{0 < t_1 < \dots < t_{r_h-1} < t_{r_h}} \int \frac{(r_h - 1)!}{(t_{r_h})^{r_h-1}} u \left(\sum_{j=1}^{r_h} \delta^{\lceil t_j/d \rceil} + \delta^{\lceil t_{r_h}/d \rceil} \right. \\ &\quad \left. \lambda_1 P_{ba} (d \lceil t_{r_h}/d \rceil - t_{r_h}) + \lambda_2 P_{br} d \frac{\delta^{\lceil t_{r_h}/d \rceil + 1}}{1 - \delta} \right) dt_1 \dots dt_{r_h-1}. \end{aligned}$$

We elaborate more on computing $E[\widehat{G}_s | t_1, \dots, t_{r_h-1}, t_{r_h}]$. The transactions arriving within time 0 to time t_{r_h} contribute $\sum_{j=1}^{r_h} u \delta^{\lceil t_j/d \rceil}$ to the long term profit gains. Consider the transactions arriving within time t_{r_h} to $d \lceil t_{r_h}/d \rceil$. This time interval belongs to time slot $\lfloor t_{r_h}/d \rfloor$. Note that in this time slot the reputation score is lower than r_h . This means that the number of transactions arriving in this time interval follows a Poisson distribution with parameter $\lambda_1 P_{ba} (d \lceil t_{r_h}/d \rceil - t_{r_h})$. In expectation, transactions arriving in this time interval contribute $\delta^{\lceil t_{r_h}/d \rceil} \lambda_1 P_{ba} (d \lceil t_{r_h}/d \rceil - t_{r_h}) u$. Consider transactions arriving from time slots $\lceil t_{r_h}/d \rceil$ to ∞ . In these time slots, a seller's reputation satisfies the condition to be labeled as reputable. The number of transactions arriving in each of these time slots follows a Poisson distribution with parameter $\lambda_2 P_{br} d$. These transactions, in expectation, contribute $u \lambda_2 P_{br} d \frac{\delta^{\lceil t_{r_h}/d \rceil + 1}}{1 - \delta}$ in total. Summing these terms together we obtain $E[\widehat{G}_s | t_1, \dots, t_{r_h-1}, t_{r_h}]$. We have

$$\begin{aligned} &\Pr[T_r \leq T_w] E[\widehat{G}_s | T_r \leq T_w] \\ &= \int_0^{T_w} \lambda_1 P_{ba} e^{-\lambda_1 P_{ba} t_{r_h}} \frac{(\lambda_1 P_{ba} t_{r_h})^{r_h-1}}{(r_h - 1)!} dt_{r_h} \\ &\quad \int_{0 < t_1 < \dots < t_{r_h-1} < t_{r_h}} \int \frac{(r_h - 1)!}{(t_{r_h})^{r_h-1}} u \left(\sum_{j=1}^{r_h} \delta^{\lceil t_j/d \rceil} + \frac{\delta^{\lceil t_{r_h}/d \rceil + 1}}{1 - \delta} \right. \\ &\quad \left. \lambda_2 P_{br} d + \delta^{\lceil t_{r_h}/d \rceil} \lambda_1 P_{ba} (d \lceil t_{r_h}/d \rceil - t_{r_h}) \right) dt_1 \dots dt_{r_h-1} \end{aligned}$$

Combing them all, we prove this theorem. ■

Proof of Theorem 5.1: We first derive $E[T_r^I]$ and P_d^I . Note that sellers advertise honestly. This means that all transactions result in positive feedbacks. This implies that the insurance certificate expires at the end of the duration time. Note that $N(\ell)$, the number of transactions at time slot $\ell = 0, 1, \dots, \infty$ before a seller ramps up, follows a Poisson distribution, and we denote its parameter by $\tilde{\lambda}_T(\ell)$. Applying Equation (9), we have $\tilde{\lambda}_T(\ell) = \lambda_2 P_{br} d$ for all $\ell = 0, 1, \dots, \lfloor T_d/d \rfloor - 1$, and $\tilde{\lambda}_T(\lfloor T_d/d \rfloor) = \lambda_2 P_{br} (T_d - d \lfloor T_d/d \rfloor) + \lambda_1 P_{ba} (d \lfloor T_d/d \rfloor + d - T_d)$, and $\tilde{\lambda}_T(\ell) = \lambda_1 P_{ba} d$ for all $\ell = \lfloor T_d/d \rfloor + 1, \dots, \infty$. Then with a similar derivation as Theorem 4.1 we obtain the expected ramp up time $E[T_r^I]$. Furthermore, with a similar derivation as Theorem 4.2 we obtain P_d^I .

Let us now derive long term profit gains (G_s) and average per seller transaction gains (G_e). We derive G_s first. Let $\widehat{G}_s^I = \sum_{\tau=0}^{\infty} \delta^{\tau} u N^I(\tau)$, where $N^I(\tau)$ denotes the number of transactions arriving in time slot τ when a seller subscribes to an insurance. Then \widehat{G}_s^I is a random variable and $G_s^I = E[\widehat{G}_s^I]$. With a similar derivation as Theorem 4.3, we have

$$\begin{aligned} E[\widehat{G}_s^I] &= \Pr[T_r^I > T_w] E[\widehat{G}_s^I | T_r^I > T_w] \\ &\quad + \Pr[T_r^I \leq T_w] E[\widehat{G}_s^I | T_r^I \leq T_w] \end{aligned}$$

Let $\widehat{\lambda}_T(t)$ denote the transaction's arrival rate at time $t[0, \infty)$. Applying Equation (9), we have two cases: (1) a seller ramps up ($T_r^I \leq T_w$), then we have $\widehat{\lambda}_T(t) = \lambda_2 P_{br}$ for all $t \in [0, \min\{T_w, T_d\}]$, $\widehat{\lambda}_T(t) = \lambda_1 P_{ba}$ for all $t \in [T_d, T_r^I]$, and $\widehat{\lambda}_T(t) = \lambda_2 P_{br}$ for all $t \in [T_r^I, \infty]$; (2) a seller drops out ($T_r^I > T_w$), then we have $\widehat{\lambda}_T(t) = \lambda_2 P_{br}$ for all $t \in [0, \min\{T_w, T_d\}]$, $\widehat{\lambda}_T(t) = \lambda_1 P_{ba}$ for all $t \in [T_d, T_w]$, and $\widehat{\lambda}_T(t) = 0$ for all $t \in [T_w, \infty]$. With these observations and using a similar derivation as Theorem 4.3, one can easily obtain analytical expressions for the term $\Pr[T_r^I > T_w] E[\widehat{G}_s^I | T_r^I > T_w]$ and the term $\Pr[T_r^I \leq T_w] E[\widehat{G}_s^I | T_r^I \leq T_w]$ respectively. Combing them all we complete this theorem. ■

Proof of Theorem 5.2: We want to derive the reasonable price that an E-Commerce operator can charge for the insurance. The marginal long term profit gain of an insured seller is $G_s^I - C_I$. Note that the marginal long

term profit gain without insurance is G_s . Thus sellers has the incentive to buy an insurance if the marginal profit gain corresponds to buying an insurance is larger than the marginal profit gain without insurance, i.e., $G_s^I - C_I > G_s$, which yields $C_I < G_s^I - G_s$.

Note that sellers advertise honestly. Let $N'(T_d)$ denote the total number of products sold in insurance duration time. It is easy to see that $N'(T_d)$ follows a Poisson distribution with parameter $\lambda_2 P_{br} T_d$. The worst case is that all buyers hold the product till the last minute of the clearing time T_c and then return it. Using a Chernoff bound [18] argument, one can easily bound the shipment cost (at the worst case) as

$$\Pr[N'(T_d)C_S \geq N'C_S] \leq e^{-\lambda_2 P_{br} T_d} (e\lambda_2 P_{br} T_d)^{N'}/N'^{N'}$$

Setting $N' = \max\{\ln \epsilon^{-1} - \lambda_2 P_{br} T_d, e^2 \lambda_2 P_{br} T_d\}$ we have $\Pr[N'(T_d)C_S \geq N'C_S] \leq \epsilon$. The clearing time follows the shipment delay d . ■